Problems for chapter 2, part 2/2

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2.13
This chapter is still too serious. Have your AI agent explain and summarize §2.7.2 on the Bernoulli equation, like a big, wet and muddy, enthusiastic dog who has just entered your living room and can't wait for dinner. Make sure the dog lists all five restrictions. Read the final result out loud; otherwise, it does not count.

2.14
In what unit(s) is the energy balance equation (equation 2/20) expressed?

- kilogram per second
- kilogram
- joule
- kilogram-second per metre
- pascal
- watt
- joule per kilogram
- watt per kilogram
- joule per second
- newton

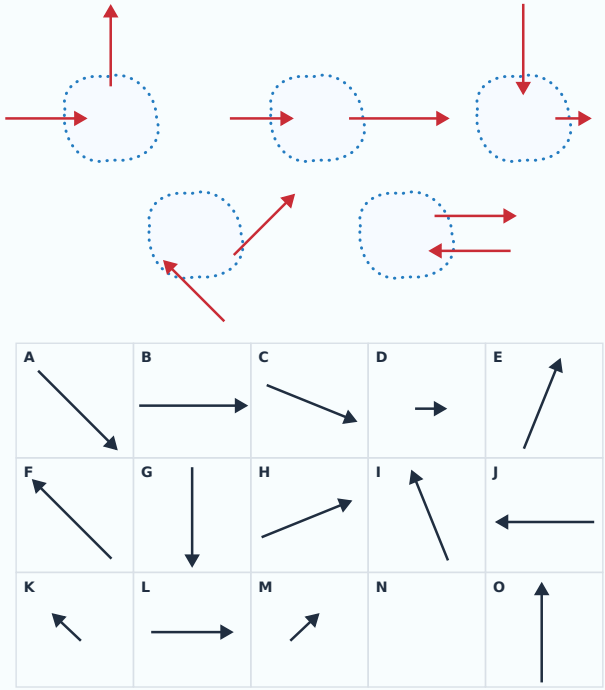
2.15
Which (if any) of the following are true about the momentum equation (equation 2/7)?

- The mass flow through the inlets must be equal to the mass flow through the outlets for it to apply.
- It relates changes in pressure to changes in velocity.
- It is only true when the energy of the system remains constant.
- It applies the net force on the control volume, and then the momentum flows out of the system.
- It quantifies the net flow of momentum through the considered volume boundaries as a vector.
- It is only true when the control system has a, uhh, what was it again? No wait. When the control volume is closed.
- It relates the energy of the system to the net flow of momentum through the considered volume boundaries.
- It states that the sum of forces on the system must always add up to zero.

2.16
Which (if any) of the following are true about the energy balance equation (equation 2/20)?

- It can only be used when there are no losses.
- It works with liquids, but not with gases.
- It cannot be applied when there is a pump or a compressor in the control volume.
- It is only valid when changes in internal energy (i) are relatively small.
- It applies only when the mass flow through the inlets compensates for the mass flow through the outlets.
- It is a vector equation with three dimensions (in x, y, and z).

2.17
Here are five control volumes (in blue) with incoming and outgoing mass flows plotted as red arrows. Which vector best describes the net force applying on the fluid for each case?



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2.18

A pipe with diameter 30 mm has a bend with angle $\theta = 130\,^\circ$, as shown in Figure 2.2. Water enters and leaves the pipe with the same speed $V_1 = V_2 = 1{,}5\ \mathrm{m\,s^{-1}}$. The velocity distribution at both inlet and outlet is uniform.

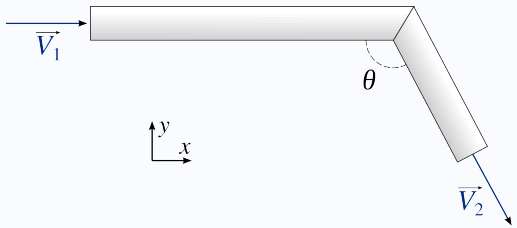


Figure 2.2: A pipe bend, through which water is flowing.

Figure CC-0 OC

2.18.1 What is the mass flow rate through the pipe?
2.18.2 What is the force exerted by the pipe bend on the water?
2.18.3 Represent the force vector qualitatively (i.e. without numerical data).
2.18.4 What would be the new force if all of the speeds were doubled?

2.19

A deflector is used behind a stationary aircraft during ground testing of a jet engine (Figure 2.3).

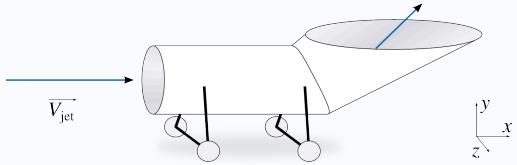


Figure 2.3: A mobile exhaust gas deflector, used to deflect hot jet engine exhaust gases upwards during ground tests.

Figure CC-0 OC

The deflector is fed with a horizontal jet of hot gases with speed $V_{jet} = 600\ \mathrm{km\,h^{-1}}$ and temperature 400 °C. Gases are discharged at an angle of 40 ° relative to the horizontal. The inlet diameter is 1 m and the outlet horizontal area is 6 $\mathrm{m^2}$.
We will assume simple conditions: uniform velocity profiles, atmospheric pressure 1 bar everywhere, no heat losses, and constant density. We will model gases with the ideal gas model: $R = 287\ \mathrm{J\,kg^{-1}\,K^{-1}}$.

2.19.1 What is the force exerted on the ground by the deflection of the exhaust gases?
2.19.2 Describe qualitatively (i.e. without numerical data) a modification to the deflector that would reduce the horizontal component of force.
2.19.3 What would the force be if the deflector traveled rearwards (positive x-direction) with a velocity of 10 $\mathrm{m\,s^{-1}}$?

2.20

A water turbine is modeled as the following system: a water jet exiting a stationary nozzle hits a blade which is mounted on a rotor (Figure 2.4). In the ideal case, viscous effects can be neglected, and the water jet is deflected through 180 °.

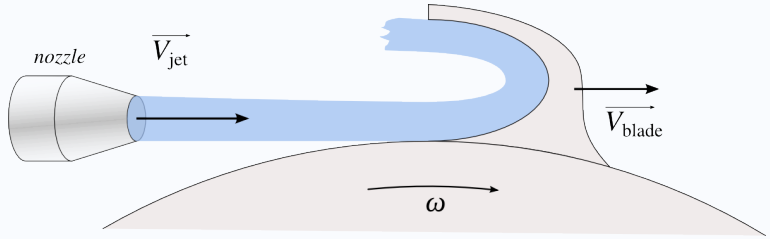


Figure 2.4: Schematic drawing of a water turbine blade. This type of turbine is called a Pelton turbine. The setup in this diagram is for clarity (it would make for a terrible turbine); in practice the water is discharged to the sides.

Figure CC-0 OC

The nozzle has a cross-sectional diameter of 5 cm and produces a water jet with a speed $V_{jet} = 15\ \mathrm{m\,s^{-1}}$. The rotor diameter is 2 m and the blade height is negligibly small.
We first study the case in which the rotor is prevented from rotating, so that the blade is stationary ($V_{blade} = 0\ \mathrm{m\,s^{-1}}$).

2.20.1 What is the force exerted by the water on the blade?
2.20.2 What is the moment exerted by the blade around the rotor axis?
2.20.3 What is the power transmitted to the rotor by the blade?

We now let the rotor rotate freely. Friction losses are negligible, and it accelerates until it reaches maximum velocity.

2.20.4 What is the rotor rotation speed?
2.20.5 What is the power transmitted to the rotor by the blade?

The rotor is now coupled to an electrical generator.

2.20.6 Show that the maximum power that can be transmitted to the rotor by the single blade occurs for $V_{blade} = \frac{1}{3} V_{water}$.
2.20.7 What is the maximum power that can be transmitted to the rotor by the single blade?
2.20.8 How would the above result change if viscous effects were taken into account? (Briefly justify your answer, e.g. in 30 words or fewer.)

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2.21

A snow plow (Figure 2.5) is clearing snow on a flat surface. We wish to quantify the power required for its operation.

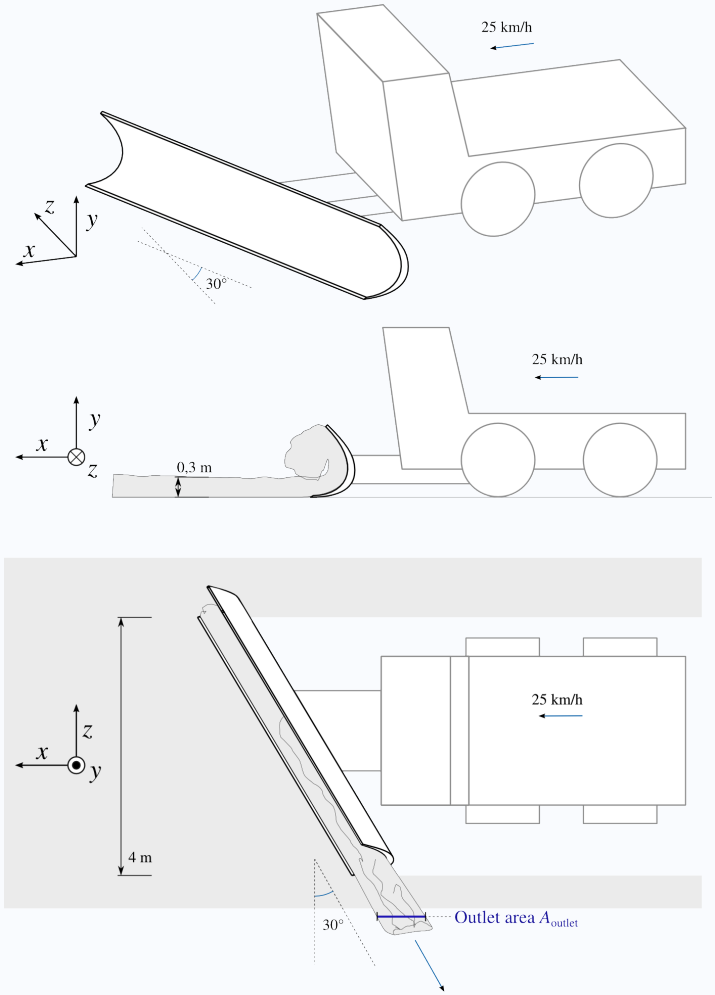


Figure 2.5: Outline schematic of a blade snow plow. The angles are measured relative to the plow.
This problem is derived from one given in Gerhart & Gross [2]. Figure CC-0 OC

The snow plow is advancing at 25 km/h. Its blade has a frontal-view width of 4 m.
The snow on the ground is 30 cm deep and has a density of 300 $\mathrm{kg\,m^{-3}}$.
The snow is pushed along the blade and is discharged horizontally at an angle of 30 ° to the left of the plow. Its density has then risen to 450 $\mathrm{kg\,m^{-3}}$. The cross-sectional area A_{outlet} of the outflowing snow in the x-y plane is 1,1 $\mathrm{m^2}$.

2.21.1 What is the force exerted on the blade by the deflection of the snow? (Indicate its magnitude and components.)

2.21.2 What is the power required for the operation of the snow plow?

2.21.3 If the plow velocity were increased by 10 %, what would be the increase in power?

2.22

A cylindrical tube is positioned perpendicular to the air flow in a wind tunnel (Figure 2.6), and the longitudinal component of velocity is measured across the tunnel section downstream of it.



Figure 2.6: A cylinder profile set up in a wind tunnel, with the air flowing from left to right.
Figure CC-0 OC

Upstream of the cylinder, the air flow velocity is uniform ($u_1 = U = 30\ \mathrm{m\,s^{-1}}$).
Downstream of the cylinder, the speed is measured across a 2 m height interval. Horizontal speed measurements are gathered and modeled with the following relationship (in $\mathrm{m\,s^{-1}}$)

$$u_{2(y)} = 29 + y^2 \tag{2/1}$$

The span of the cylinder (perpendicular to the flow) is 2 m. The Mach number is very low, and the air density remains constant at $\rho = 1{,}23\ \mathrm{kg\,m^{-3}}$; pressure is the same upstream and downstream.

2.22.1 What is the drag force acting on the cylinder?

2.22.2 How would this value change if the flow in the cylinder wake were turbulent, and the function $u_{2(y)}$ above only modeled *time-averaged values* of the horizontal velocity? (Briefly justify your answer, e.g. in 30 words or fewer.)

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2.23

We wish to measure the drag acting on a thin plate positioned parallel to an air stream. In order to achieve this, measurements of the horizontal velocity u are made around the plate (Figure 2.7).

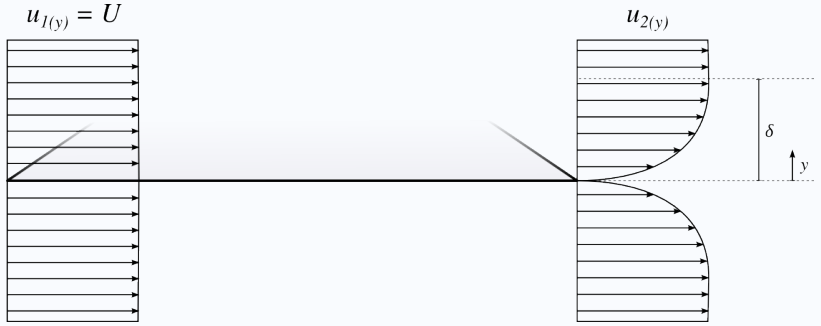


Figure 2.7: Side view of a plate positioned parallel to the flow.

Figure CC-0 OC

At the leading edge of the plate, the horizontal velocity of the air is uniform: $u_1 = U = 10\ \mathrm{m\,s^{-1}}$. At the trailing edge of the plate, we observe that a thin layer of air has been slowed down by the effect of shear. This layer, called a *boundary layer*, has a thickness of $\delta = 1\ \mathrm{cm}$. The horizontal velocity profile can be modeled with the relation:

$$u_{2(y)} = U\left(\frac{y}{\delta}\right)^{\frac{1}{7}} \qquad (2/2)$$

The width of the plate (perpendicular to the flow) is 30 cm and it has negligible thickness. The flow is incompressible ($\rho = 1{,}23\ \mathrm{kg\,m^{-3}}$) and the pressure is the same upstream and downstream.

2.23.1 What is the drag force acting on the plate?

2.23.2 What is the power dissipated by the air through shear?

2.23.3 The air clearly loses kinetic energy while passing over the plate. Where has this energy gone, and how could it be measured in its new form? (Briefly justify your answer, e.g. in 30 words or fewer.)

2.24

A group of students takes airspeed measurements in a wind tunnel. The objective is to relate the velocity of the air to the drag acting on a wing profile positioned across the tunnel test section (Figure 2.8).



Figure 2.8: Wing profile positioned across a wind tunnel. The horizontal velocity distributions upstream and downstream of the profile are also shown.

Figure CC-0 OC

Upstream of the profile, the air flow velocity is uniform ($u_1 = U = 50\ \mathrm{m\,s^{-1}}$).
Downstream of the profile, horizontal velocity measurements are made every 5 cm across the flow; the following results are obtained:

Vertical position (cm)	Horizontal speed u_2 ($\mathrm{m\,s^{-1}}$)
0	50
5	50
10	49
15	48
20	45
25	41
30	39
35	40
40	43
45	47
50	48
55	50
60	50

The span of the profile (perpendicular to the flow) is 50 cm. The airflow is incompressible ($\rho = 1{,}23\ \mathrm{kg\,m^{-3}}$) and the pressure is the same upstream and downstream.

2.24.1 What is the drag acting on the profile?

2.24.2 How would the above calculation change if *vertical* speed measurements were also taken into account?

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2.25 Optional: moment on deflector

We revisit the exhaust gas deflector of Problem 2.19. Figure 2.9 below shows the deflector viewed from the side. The midpoint of the inlet is 2 m above and 5 m behind the wheel labeled "**A**", while the perpendicular distance to the outlet velocity line of action is 1,72 m.

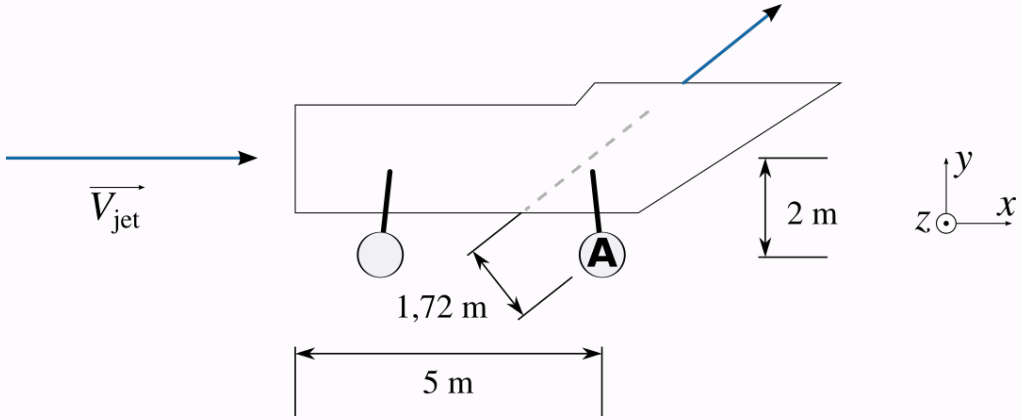


Figure 2.9: Side view of the mobile exhaust gas deflector which was shown in Figure 2.3

Figure CC-0 OC

What is the moment generated by the gas flow about the axis of the wheel labeled "A"?

2.26 Optional: helicopter tail

In a helicopter, the role of the tail is to counter exactly the moment exerted by the main rotor about the main rotor axis. This is usually done using a tail rotor which is rotating around a horizontal axis.

A helicopter, shown in fig. 2.10, is designed to use a tail *without* a rotor, to reduce the risk of accidents when landing and taking off. The tail is a long hollow cylindrical tube with two inlets and one outlet. To simplify calculations, we consider that pressure is atmospheric at every inlet and outlet.

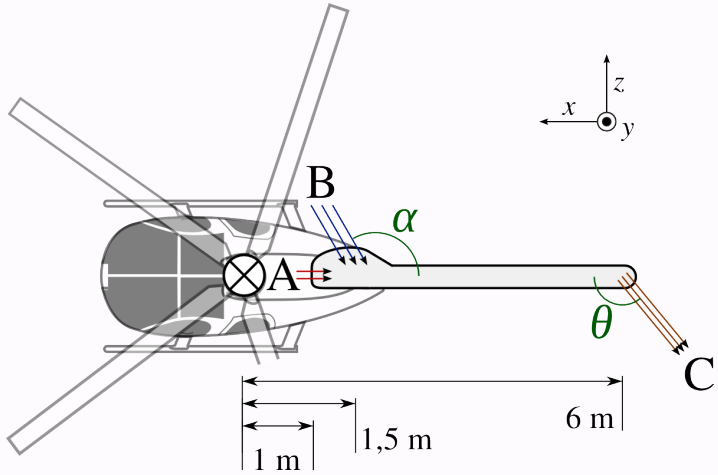


Figure 2.10: Top view of a helicopter using a rotorless tail.

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- Inlet A has a cross-sectional area of 0,2 m^2. It contributes hot exhaust gases of density 0,8 $kg\,m^{-3}$ and velocity 12 $m\,s^{-1}$, aligned with the (x) axis of the tail;
- Inlet B contributes 25 $kg\,s^{-1}$ of atmospheric air incoming at an angle α = 130 ° relative to the axis of the tail, with a velocity of 3 $m\,s^{-1}$.

The mixture of exhaust gases and atmospheric air is discharged at the tip of the tail (outlet C) with a fixed velocity of 45 $m\,s^{-1}$. The angle θ at which gases are discharged is controlled by the flight computer.

2.26.1 What is the discharge angle θ required so that the tail generates a moment of +6 kN m around the main rotor (y) axis?

2.26.2 Propose and quantify a modification to the tail geometry or operating conditions that would allow the tail to produce no thrust (that is to say, zero force along the x-axis), while still generating the same moment.

This system is commercialized by MD Helicopters as the NOTAR. The original design used exhaust gases like in this problem; but this feature was abandoned and the air is now moved with a fan. However, a clever use of air circulation around the tail pipe axis, called the Coandă effect[w], contributes to the generated moment.

2.27 Optional: do it with AI: snow plow parameters

Revisit Problem 2.21 with the help of an interactive diagram. Plot force and power as a function of velocity, with input sliders for the blade angle and density of the discharged snow.

2.28 Optional: do it with AI: flow over a hill

Download the flow case at https://learnfluidmechanics.com/case069_student_export.zip and explore the data. This is a high-quality measurement of air flow over a 2D hill ("ERCOFTAC case 069" [1, 13]) used as reference data for calibrating computations. Treat it like a scientist or a customer: with healthy suspicion and curiosity. Does it make sense, and are there flaws?

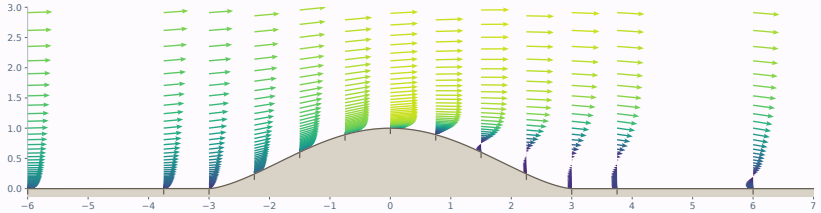


Figure 2.11: up and over the hill, wee!
Data from ERCOFTAC case 069 [1, 13]

2.28.1 Plot the velocity profiles at all the available locations, upstream, above the hill, and downstream of it.

2.28.2 Plot the velocity vectors, colored according to their magnitude, in a region close to the hill, making it clearly visible that the flow is backwards in the wake of the hill.

2.28.3 Calculate the mass flow rate (per unit span) through each measurement plane, and plot that as a function of streamwise distance. Think carefully about the result. How come this is not a flat line? (Hint: we have dealt with this very issue in Problem 2.22 and siblings.)

Now, we want to quantify momentum flows.

2.28.4 Calculate the mass flow rate at the third measurement plane, and then calculate the height corresponding to the same mass flow rate for each of the other measurement planes. This should give you the height of a streamtube for the flow, which you can plot with respect to streamwise distance.

2.28.5 Within this streamtube, calculate the flow of momentum (per unit span) at each cross-section, and plot the results as a function of streamwise distance. In which direction does the net force on the flow point?

Gather here the equations you would like to have provided on an equation sheet for the exam, and share them in class

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Answers

2.14.1 joule per second and watt (same thing)

2.15.1 it quantifies the net flow of momentum as a vector

2.16.1 wrong! they are all wrong!

2.17.1 F, D, E, L and B. Hey look, it spells out the acronym for *Fluid Dynamics: Enjoy Lovely Bubbles*!

2.18.1 $\dot{\vec{m}} = 1{,}0603\ \mathrm{kg\ s^{-1}}$

2.18.2 $\vec{F}_{\mathrm{net}} = (-0{,}5681 - 1{,}2184)\ \mathrm{N}$ (sum of forces due to gravity, pressure, and shear)

2.18.4 The force is quadrupled.

2.19.1 Force on the fluid is $\vec{F}_{\mathrm{net}} = (-9{,}532 + 1{,}479)\ \mathrm{kN} : \|\vec{F}_{\mathrm{net}}\| = 9{,}646\ \mathrm{kN}$ (force on ground is opposite: $\vec{F}_{\mathrm{fluid\ on\ pipe}} = -\vec{F}_{\mathrm{pipe\ on\ fluid}}$);

2.19.3 $\|\vec{F}_{\mathrm{net\ 2}}\| = 8{,}525\ \mathrm{kN}$.

2.20.1 $F_{\mathrm{net}} = (-883{,}60)\ \mathrm{N}$;

2.20.2 $M_{\mathrm{net},X} = |F_{\mathrm{net}}|\ R = 883{,}6\ \mathrm{N\,m}$;

2.20.3 $\dot{W}_{\mathrm{rotor}} = 0\ \mathrm{W}$;

2.20.4 $\omega = 143{,}2\ \mathrm{rpm}$ ($F_{\mathrm{net}} = 0\ \mathrm{N}$);

2.20.5 $\dot{W}_{\mathrm{rotor}} = 0\ \mathrm{W}$ again;

2.20.7 $\dot{W}_{\mathrm{rotor,\ max}} = 1{,}963\ \mathrm{kW}$ @ $V_{\mathrm{blade,\ optimal}} = \frac{1}{3} V_{\mathrm{water\ jet}}$.

2.21.1 $\dot{m} = 2500\ \mathrm{kg\ s^{-1}}$, $F_{\mathrm{net}\,x} = +10{,}07\ \mathrm{kN}$, $F_{\mathrm{net}\,z} = -12{,}63\ \mathrm{kN}$ (force on blade is opposite);

2.21.2 $\dot{W} = \vec{F}_{\mathrm{net}} \cdot \vec{V}_{\mathrm{plow}} = F_{\mathrm{net}\,x}\,|V_1| = 69{,}94\ \mathrm{kW}$;

2.21.3 $\dot{W}_2 = 1{,}1^3\,\dot{W}$ (+33 %).

2.22.1 $F_{\mathrm{net}\,x} = \rho L \int_{S2} \left(u_2^2 - U u_2\right)\,\mathrm{d}y = -95{,}78\ \mathrm{N}$.

2.23.1 For both faces, the force on the fluid is $F_{\mathrm{net}\,x} = 2\rho L \int_0^\delta \left(u_{(y)}^2 - U u_{(y)}\right)\,\mathrm{d}y = -7{,}175 \cdot 10^{-2}\ \mathrm{N}$; force on plate is opposite.

2.23.2 $\dot{W}_{\mathrm{drag}} = U\,|F_{\mathrm{net}\,x}| = 0{,}718\ \mathrm{W}$.

2.24.1 Force on fluid is $F_{\mathrm{net}\,x} \approx \rho L \sum \left[\left(u_2^2 - U u_2\right)\,\delta y\right] = -64{,}8\ \mathrm{N}$ (force on wing is opposite)

2.25.1 Re-use $\dot{m} = 67{,}76\ \mathrm{kg\ s^{-1}}$, $V_1 = 166{,}7\ \mathrm{m\ s^{-1}}$, $V_2 = 33{,}94\ \mathrm{m\ s^{-1}}$ from Problem 2.19. With $R_{2 \perp V_2} = 1{,}717\ \mathrm{m}$, plug in numbers in eq. 2/13: $M_{\mathrm{net}} = +18{,}64\ \mathrm{kN\,m}$ in z-direction; moment on deflector is opposite.

2.26.1 Work eq. 2/13 down to scalar equation (in y-direction), solve for θ: $\theta = 123{,}1\ ^\circ$.

2.26.2 There are multiple solutions which allow both moment and force equations to be solved at the same time. r_C can be shortened, the flow in C can be split into forward and rearward components, or tilted downwards etc. Reductions in $\dot{m}_B$ or V_C are also possible, but quantifying them requires solving both equations at once.

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