Problems for chapter 2, part 1/2

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2.1
This chapter is much too serious. Have your AI agent explain and summarize equation 2/7 and §2.4.2 on the balance of momentum, like a conspirationist who is trying to to convince you that "they" don't want you to know all of this. Read the final result out loud, otherwise it does not count.

2.2
In which of the following cases can we ABSOLUTELY NOT use the Bernoulli equation (equation 2/22)?

- When there is heat transfer
- When the pressure is changing
- When the velocity is too low
- When the altitude remains constant
- When the density is changing
- When the flow is unsteady

2.3
In what unit is the mass balance equation (equation 2/3) expressed?

- kilogram
- Watt per kilogram
- Joule
- none
- kilogram per second
- kilogram-second per metre
- Pascal per second
- Watt
- Joule per second
- In a special secret unit which is shared only among fluid dynamicists

2.4
What does the mass balance equation (equation 2/3) state?

- If the area is decreased, the velocity is increased.
- The density can only change if the area is changed.
- None of the other answers.
- All of the answers except "None of the other answers".
- The pressure can only change if the area is changed.
- The multiple of pressure, area, and velocity must remain constant.
- If the area is decreased, the velocity is decreased.
- The velocity can only increase if the pressure is decreased.

2.5
Water flows from left to right in a pipe. On the left, the diameter is 8 cm, the water arrives with a uniform velocity of 1,5 $\mathrm{m\,s^{-1}}$. The diameter increases gently until it reaches 16 cm. The expansion is smooth, and this is the first problem in the sheet, so we consider that losses (specifically, energy losses due to wall friction and flow separation) are negligible.

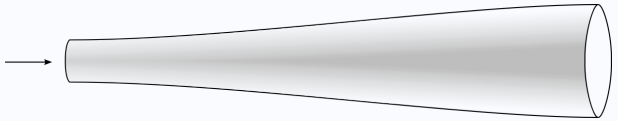


Figure 2.1: A simple pipe expansion, with water flowing from left to right.
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2.5.1 What are the mass and volume flows at inlet and outlet?
2.5.2 What is the average velocity of the water at the right end of the expansion?
2.5.3 What is the pressure change in the water across the expansion?
The volume flow of water in the pipe is now doubled.
2.5.4 What is the new pressure change?
The water is drained from the pipe, and instead, air with density 1,225 $\mathrm{kg\,m^{-3}}$ is flowed in the pipe, incoming with a uniform velocity of 1,5 $\mathrm{m\,s^{-1}}$.
2.5.5 What are the new mass and volume flows?
2.5.6 What is the new pressure change?

2.6
Water flows in a long pipe which has constant diameter; a valve is installed in the middle of the pipe length. Water arrives the pipe with a uniform velocity of 1,5 $\mathrm{m\,s^{-1}}$ and the pipe diameter is 250 mm.

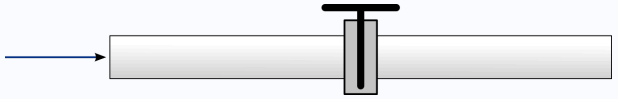


Figure 2.2: A simple, straight pipe, featuring a partially-open valve in the center
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The pipe itself and the valve, together, induce a pressure loss which can be quantified using the dimensionless *loss coefficient* K_{valve} (we later will later encounter it in chapter 5). With this tool, the pressure loss Δp_{valve} is related to the mean incoming speed V_{incoming} as:

$$K_{\text{valve}} \equiv \frac{|\Delta p_{\text{valve}}|}{\frac{1}{2}\rho V_{\text{incoming}}^2} = 2{,}6 \qquad (2/1)$$

2.6.1 What is the outlet velocity of the water?
(note: this is a classic "trick" question! :-)
2.6.2 What is the drop in water pressure across the valve?
2.6.3 What is the power dissipated into internal energy?
2.6.4 If the heat losses of the pipe and valve are negligible, what is the temperature increase of the water?

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2.7

A jet engine is equipped with several *combustors* (sometimes also called *combustion chambers*). We are interested in fluid flow through one such combustor, shown in fig. Figure 2.3. Air from the compressor enters the combustor, is mixed with fuel, and combustion occurs, which greatly increases the temperature and specific volume of the mix, before it is run through the turbine.

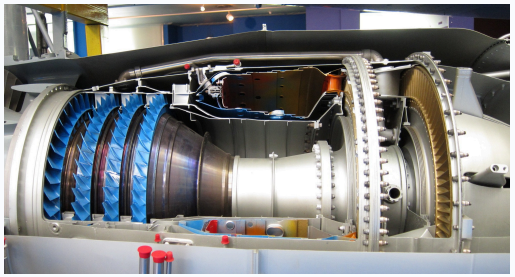


Figure 2.3: A combustor in a sectioned jet engine (here, a Turboméca Adour). Air enters from the left, out of the compressor (whose blades are painted blue). It leaves the combustor on the right side, into the turbine. In the combustor, high-temperature, steady combustion takes place.

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The conditions at inlet are as follows:

- Air mass flow: 0,5 $\mathrm{kg\,s^{-1}}$
- Air properties: 25 bar, 1050 °C, 12 $\mathrm{m\,s^{-1}}$
- Fuel mass flow: 5 $\mathrm{g\,s^{-1}}$.

At the outlet, the hot gases have pressure 24,5 bar and temperature 1550 °C, and exit with a speed of 50 $\mathrm{m\,s^{-1}}$.

We assume that the incoming air and outgoing gas have the same thermodynamic properties: $c_v = 718\ \mathrm{J\,kg^{-1}\,K^{-1}}$, $R_{air} = 287\ \mathrm{J\,kg^{-1}\,K^{-1}}$.

2.7.1 What are the mass flows at inlet and outlet?

2.7.2 What are the volume flows at inlet and at outlet?

2.7.3 What is the power provided to the flow as heat?

2.7.4 What is the net force exerted on the gas as it travels through the combustor?

2.8

A pipe with a diameter of 1 m is used to take water from a tank.

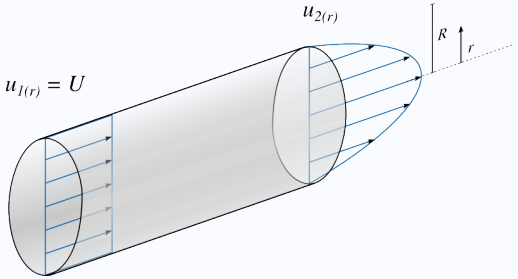


Figure 2.4: Velocity profiles at the inlet and outlet of a circular pipe.

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At the entrance of the pipe, the speed is uniform: $u_1 = U_{av.} = 5\ \mathrm{m\,s^{-1}}$.

Shear applies on the fluid from the walls. This strains the fluid particles, and changes the velocity distribution. At the outlet of the pipe, the velocity profile is no longer uniform. It can be modeled as a function of the radius with the relationship:

$$u_{2(r)} = U_{\text{center}}\left(1 - \frac{r}{R}\right)^{\frac{1}{7}} \tag{2/2}$$

What is the center velocity U_{center} at the outlet?

2.9

A retired fluid dynamicist is watering plants, sees an Amazon delivery van parked in front of their garden, and tries something fun. They point the hose nozzle so it shoots water towards the back the van. (This is how fluid dynamicists understand "fun".)

The nozzle has a 3 cm^2 cross-sectional area, and the water speed at the nozzle outlet is $V_{\text{jet}} = 20\ \mathrm{m\,s^{-1}}$. As the horizontal water jet hits the back of the van, it is split in two symmetrical vertical flows (Figure 2.5). The two opposite vertical jets have same mass flow and same velocity ($V_2 = V_3 = 20\ \mathrm{m\,s^{-1}}$).

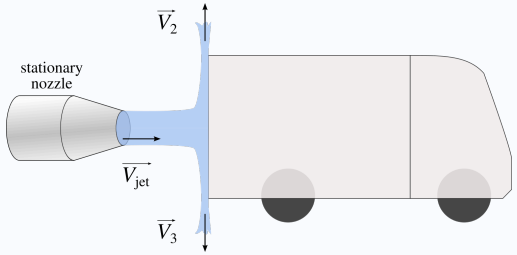


Figure 2.5: A water jet flowing out of a nozzle (left), and impacting the vertical back surface of a small electric van, on the right.

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2.9.1 What is the net force exerted on the water by the van?

2.9.2 What is the net force exerted on the van by the water?

Now, the driver speeds away, and the van moves longitudinally in the same direction as the water jet, with a speed $V_{\text{van}} = 15\ \mathrm{m\,s^{-1}}$. (As you may have guessed, this is a conceptual setup, which allows us to approach conceptually the case where water acts on the blades of a turbine.)

2.9.3 What is the new force exerted by the water on the van?

2.9.4 What is the mechanical power transmitted to the van?

2.9.5 How would the power be modified if the volume flow was kept constant, but the diameter of the nozzle was reduced? (briefly justify your answer, e.g. in 30 words or less)

2.10

Researchers build a high-speed wind tunnel. For this, they build a large compressed air tank. Air escapes from the tank into a pipe which decreasing cross-section, as shown in Figure 2.6. The pipe diameter reaches a minimum (at the tunnel *throat*), and then it expands again, before discharging into the atmosphere outside.

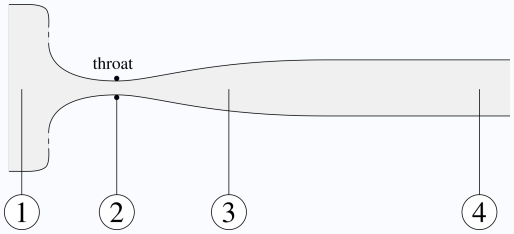


Figure 2.6: A converging-diverging nozzle. Air flows from the left tank to the right outlet, with a contraction in the middle.

Figure CC-0 OC

Because the flow is complicated enough as it is, we will assume that heat losses through the tunnel walls are negligible, and that the fluid has uniformly-distributed velocity in cross-sections of the pipe.

In the tank (point 1), the air is stationary, with pressure 7,8 bar and temperature 246,6 °C.

At the throat (point 2), the pressure and temperature have dropped to 4,2 bar and 160 °C. The velocity has reached 417,2 $\mathrm{m\,s^{-1}}$. The throat cross-section is 0,01 m^2.

2.10.1 What is the mass flow through the tunnel?

2.10.2 What is the net force exerted on the fluid between the tank and the throat?

2.10.3 What is the kinetic energy per unit mass of the air at the throat?

Downstream of the throat, the pressure keeps dropping. By the time it reaches a point 3, the air has seen its pressure and temperature drop to 1,38 bar and 43 °C.

2.10.4 What is the fluid velocity at point 3?
(if you need to convince yourself that $A_3 > A_1$, you may also calculate the cross-section area)

2.10.5 What is the net force exerted on the fluid between the points 2 and 3?

2.10.6 What is the kinetic energy per unit mass of the air at point 3?

Once it has passed point 3, the air undergoes complex loss-inducing evolutions (including going through a *shock wave*, where its properties change very suddenly), before it discharges into the atmosphere (point 4) with pressure 1 bar and temperature 165 °C.

2.10.7 What is the fluid velocity at outlet?

2.10.8 What is the outlet cross-section area?

2.10.9 What is the net force exerted on the fluid between section 3 and the outlet?

2.10.10 What is the kinetic energy per unit mass of the air at the outlet?

2.10.11 Nerds will want to compare the speed of the air to the speed of sound inside the air (the speed of pressure). The comparison is made using the *Mach number*: $[\mathrm{Ma}] \equiv \frac{V_{\text{fluid}}}{V_{\text{sound}}}$. It turns out that in air, the speed of sound depends only on the local temperature: $V_{\text{sound, air}} = \sqrt{\gamma R T_{\text{air}}}$. Calculate the Mach number in the tank, at the throat, and at points 3 and 4.

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2.11 Optional: do it with AI: Optimum van escape

In problem 2.9, speed the delivery van clearly has an effect on the amount of power it receives. Plot the power as a function of the van speed, identifying the maximum.

2.12 Optional: do it with AI: Mass flows in a real flow

Download the flow case at https://learnfluidmechanics.com/case007_student_export.zip and explore the data. This is a high-quality measurement of air flow in a duct ("ERCOFTAC case 007") used as reference data for calibrating computations. Treat it like a scientist or a customer: with healthy suspicion and curiosity. Does it make sense, and are there flaws?

2.12.1 Plot the magnitude of velocity at the four cross-sections. Have the plots mirror the data across the $y = 0$ plane so you get a better view

2.12.2 Calculate mass flow at different sections. By how much do they differ and why? Plot all four values on a diagram.

2.12.3 Choose one cross-section and represent the flow *without* the axial component (the part of the flow that is across the main flow direction), with vectors or with line integral convolution.

2.12.4 Calculate the momentum flow at the first and last cross-sections and the corresponding net force applying on the air.

Answers

2.5.1 At both inlet and outlet, $\dot{m} = 7{,}53\ \mathrm{kg\ s^{-1}}$ and $\dot{\mathscr{V}} = 7{,}53\ \mathrm{L\ s^{-1}}$

2.5.2 $V_2 = 0{,}375\ \mathrm{m\ s^{-1}}$

2.5.3 $\Delta p_{1\to2} = +1054\ \mathrm{Pa}$

2.5.4 $\Delta p_{3\to4} = +4218\ \mathrm{Pa}$

2.5.5

2.5.6 $\Delta p_{5\to6} = +1{,}29\ \mathrm{Pa}$

2.6.1 $V_2 = V_1 = 1{,}5\ \mathrm{m\ s^{-1}}$ by application of the mass balance equation; although a mis-application of the energy equation would suggest otherwise

2.6.2 With equation 2/1, $\Delta p_{\mathrm{valve}} = -2925\ \mathrm{Pa}$

2.6.3 $\dot{m}\Delta i = +215{,}37\ \mathrm{W}$

2.6.4 With eq. 2/21, $\Delta T = +0{,}7\ \mathrm{mK}$ (very small!)

2.7.1 $|\dot{m}_1| = 0{,}005 + 0{,}5\ \mathrm{kg\ s^{-1}}$ and $|\dot{m}_2| = 0{,}505\ \mathrm{kg\ s^{-1}}$

2.7.2 $\dot{\mathscr{V}}_1 = 0{,}0759\ \mathrm{m^3\ s^{-1}}$ & $\dot{\mathscr{V}}_2 = 0{,}1078\ \mathrm{m^3\ s^{-1}}$ (there is no volume balance equation!)

2.7.3 $\dot{Q} = +261\ \mathrm{kW}$ (using $V_2 = 50\ \mathrm{m\ s^{-1}}$)

2.7.4 $F_{\mathrm{net}} = +19{,}25\ \mathrm{N}$ (in flow-wise direction)

2.7.5 $F_{\mathrm{net\ on\ water}} = -120\ \mathrm{N}$

2.8.1 $U_{\mathrm{center}} = \mathrm{num}\{1, 2245\} U_{\mathrm{av.}}$

2.9.1 $F_{\mathrm{water/van}} = -F_{\mathrm{net\ on\ water}}$

2.9.2 $F_{\mathrm{net\ on\ water}} = -7{,}5\ \mathrm{N}$

2.9.3 $W_{\mathrm{van}} = 112{,}5\ \mathrm{W}$

2.10.1 With eq. 2/21, $V_2 = 417{,}2\ \mathrm{m\ s^{-1}}$, and so $\dot{m}_2 = \dot{m} = 14{,}1\ \mathrm{kg\ s^{-1}}$

2.10.2 $F_{\mathrm{net\ 1\to2}} = +5{,}882\ \mathrm{kN}$ (flow-wise)

2.10.3 $e_{k2} = 87{,}03\ \mathrm{kJ\ kg^{-1}}$

2.10.4 With eq. 2/21, $V_3 = 638{,}71\ \mathrm{m\ s^{-1}}$ (you may then calculate ρ_3 and obtain $A_3 > A_2$ even though $V_3 > V_2$, a classical feature of supersonic flows)

2.10.5 $F_{\mathrm{net\ 2\to3}} = +3{,}137\ \mathrm{kN}$

2.10.6 $e_{k3} = 204{,}61\ \mathrm{kJ\ kg^{-1}}$

2.10.7 $V_4 = 405\ \mathrm{m\ s^{-1}}$

2.10.8 $\rho_4 = 0{,}7952\ \mathrm{kg\ s^{-1}}$ and so $A_4 = 0{,}0438\ \mathrm{m^2}$

2.10.9 $F_{\mathrm{net\ 3\to4}} = -3{,}309\ \mathrm{kN}$ (so, against the flow direction)

2.10.10 $e_{k4} = 82{,}01\ \mathrm{kJ\ kg^{-1}}$

2.10.11 $[\mathrm{Ma}]_1 = 0$ (zero velocity) and $[\mathrm{Ma}] = 1$ (a classical feature of compressible flow expansions). $[\mathrm{Ma}]_3 = 1{,}795$ and $[\mathrm{Ma}]_4 = 0{,}96$: something must have happened here…