

CHAPTER 1

Basic Flow Quantities

✦ Fundamental concepts required to dive into fluid mechanics

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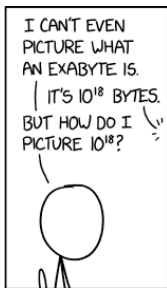
1.1 What are we doing here?

When a fluid dynamicist first starts working on a flow, in a company or in a laboratory, they rapidly develop a “feel” for how big and complicated it is. We want to think like they do. For this, we need to answer a few questions, and for that we need a few concepts in our mind. Let’s dive right in.

1.2 How to think of a fluid



Video: how to think of a fluid
by OC (STYL)
<https://youtu.be/Rh9azPIS7Dw>



XKCD #2283: how (not) to picture big numbers
by Randall Munroe (CC-BY-NC)
<https://xkcd.com/2283>

Think of a *fluid* as a type of matter that will deform endlessly, and which will very rapidly expand to occupy all of the space it has available. What stops the expansion is either a solid wall (like the lid of a container) or the presence of another fluid.

Like all matter, fluids are made of discrete, solid molecules. However, in fluid mechanics, we work at the macroscopic scale: at that scale, matter can be treated like a *continuum*, and we never see the molecules in our equations.

There are about $2 \cdot 10^{22}$ molecules in the air inside an “empty” one-liter bottle at ambient temperature and pressure. Even when the air in the bottle is completely still, these molecules are constantly colliding with each other and with the bottle walls; on average, their speed is equal to the speed of sound: approximately 1000 km h^{-1} .

Fluid flows occur far above this scale. Even the most turbulent (complex) flows can be accurately described by considering the velocities of *groups* of several millions of molecules collectively. We name these groups *fluid particles*.^w By doing so, we never find out the velocity of individual molecules: instead, those are averaged in space and time and result in much simpler and smoother trajectories, which are those we can observe with macroscopic instruments such as video cameras and pressure probes.

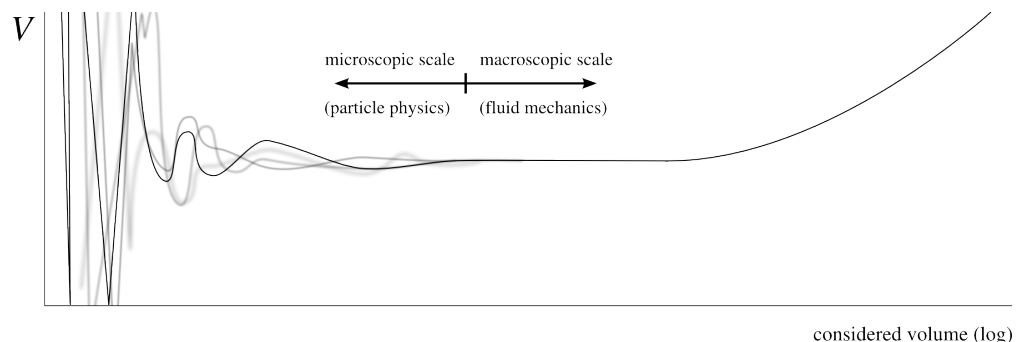


Figure 1.1: Measurement of the average value of a property (here, velocity V ; but it could be pressure, or temperature) inside a given volume. As the volume shrinks towards zero, the fluid can no longer be treated as a continuum; and property measurements will oscillate wildly.

Figure CC-BY-SA OC

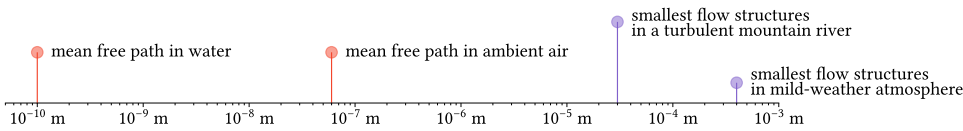


Figure 1.2: Different scales: look at a mountain river on a calm summer day. The smallest fluid flow movements are still much larger than the distance between molecules. The logarithmic scale here is very important: the lengths with the blue dots on the right are about 10000 times larger than the ones with the red dots on the left.

Figure CC-BY-NC OC

Adopting this point of view, which is named the *continuum abstraction*, is not a trivial decision, because the physical laws which determine the behavior of molecules are very different from those which determine the behavior of elements of fluid. For example, in fluid mechanics the fluid particles do not attract or repel one another; while new forces “appear” due to pressure or shear effects that do not exist at a molecular level.

A direct benefit of the continuum abstraction is that it’s a lot less work. Describing the “still air” in the bottle mentioned above, for example, requires only a single equation instead of a system of $2 \cdot 10^{22}$ equations with $2 \cdot 10^{22}$ unknowns (all averaging to exactly zero because the fluid is not moving!).

1.3 Knowledge we need from other disciplines

- From mechanics: forces, energy, work. This is because we approach fluids as mechanical objects. Meriam & Kraige [1, 2] is a good pair of books to check out if you are rusted.
- From mechanics: acceleration, velocity, position (see Figure 1.3). In general we are interested in how fluids move, so we really need these three concepts. Again refer to Meriam & Kraige.
- From thermodynamics: heat, temperature, pressure. You need a very clear understanding of these three concepts, but it’s not necessary to go into the details of the transformation processes nor of entropy. Check out the author’s free textbook at thermodynamicsbook.com [5] if needed.
- From mathematics: integrals, derivatives, vector addition and dot product. Check out Kalid Azad’s betterexplained.com to revise if needed.

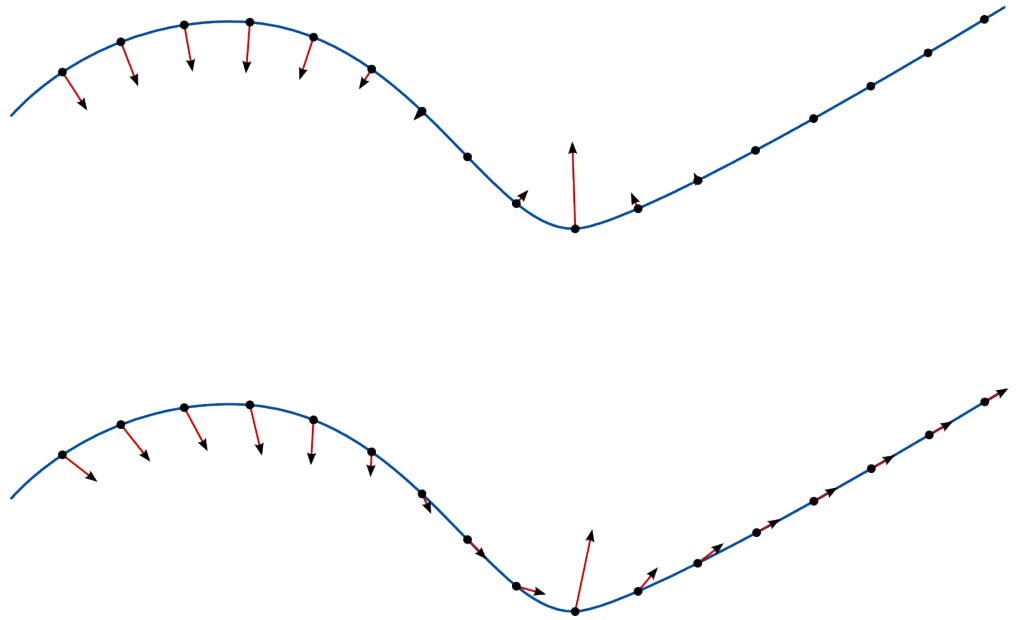


Figure 1.3: A body (black dot) is following a trajectory, plotted in blue. The acceleration vectors are plotted in red for two cases: on top, when the magnitude of its velocity remains constant, and on the bottom, when this magnitude changes continuously. Make sure you can describe acceleration as a vector, we will need this tool in fluid mechanics.

Figure CC-0 OC

1.4 Density



Video: density

by OC (STYL)
<https://youtu.be/1Jd597DXngs>

The density ρ (Greek letter rho) is the amount of mass per unit volume:

$$\rho \equiv \frac{m}{\mathcal{V}} \quad (1/1)$$

where ρ is the density (kg m^{-3});
 m is the considered mass (kg);
 and $\mathcal{V}$ is the considered volume (m^3) (note the curly $\mathcal{V}$ to set it apart from velocity V).

Advice from an expert

Never confuse pressure p , which is how hard a fluid pushes on walls, with density ρ , which is how much of the fluid there is per unit volume. Not many fluid dynamicists will confess to it, but most have, as beginner students, mixed up the two symbols once in a moment of weakness. Make sure you write those two letters very clearly, so you distinguish them easily even when stressed or distracted.



A few densities are shown in Figure 1.4. Remember two of them: at ambient conditions, air has about $\rho_{\text{air}} = 1,2 \text{ kg m}^{-3}$; and water is about $\rho_{\text{water}} = 1000 \text{ kg m}^{-3}$.

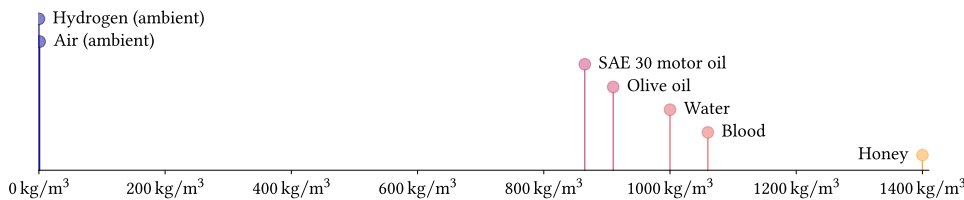


Figure 1.4: The densities of a few fluids.

Figure CC-BY-NC OC

You may remember from thermodynamics that the density of fluids *can* change, whenever they are heated or when they do work (or conversely, are cooled or have work done to them). This brings a whole level of complexity for us: the energy transfers change the density, which affects where the fluid is going, which affects the energy transfers again. When density changes in a flow, we say the flow is *compressible*. Otherwise we call it *incompressible*.

Advice from an expert

The word “incompressible” is a really mean false friend: it has nothing to do with pressure p , and all with density ρ . Most flows with moderate speeds and powers are incompressible: the density does not change.

Pressure will always change in space or even in time if there is a fluid flow, so there is no word for “pressure remains the same” in fluid mechanics.



We can often calculate density as follows: gases that have relatively simple molecules, moderate temperature and low pressure can be described with the *ideal gas model*.^w Their density is then related to their absolute temperature T and pressure p according to a single, approximately constant parameter $R_{\text{specific}} \equiv \frac{p}{\rho} T$:

$$\frac{p}{\rho} = R_{\text{specific}} T \quad (1/2)$$

where R_{specific} depends on the state and nature of the gas ($\text{J K}^{-1} \text{kg}^{-1}$);
 T is the absolute temperature (K);
 and p is the pressure (Pa).

Note that R_{specific} here is a *specific* gas constant (whose value depends on the gas); chemists often instead use a *universal* definition of R in $\text{J mol}^{-1} \text{K}^{-1}$.

This type of model (relating temperature to pressure and density) is called an *equation of state*. When R remains approximately constant, the fluid is said to behave as an ideal gas. Other models exist^w which predict the properties of gases over larger property ranges, at the cost of increased mathematical complexity. Many fluids, especially liquids, do not follow this equation and their temperature must be determined in another way, most often with the help of laboratory measurements.

For now, you may calculate or look up one value of density for the fluid of your flow, then **neglect density changes**:

- for liquids (liquids have a high bulk modulus, which means a lot of effort is required to change their density),
- for gases flowing freely without heat transfer at moderate speeds,
- and in all cases in your first approach.

We will revisit compressibility effects (density changes) later in this course.

1.5 Temperature

Temperature does not vary much in fluids, because they have high heat capacity ($c_{\text{air}} \approx 1 \text{ kJ K kg}^{-1}$, $c_{\text{water}} \approx 4 \text{ kJ K kg}^{-1}$). Unless the changes in velocity are very high, the temperature changes associated with fluid flow are much too small to be measurable in practice.

Although we can “feel” temperature in daily life, we serve as pretty terrible thermometers. This is because we constantly produce heat, and we infer temperature by the power our body loses or gains as heat. Think about it. This power:

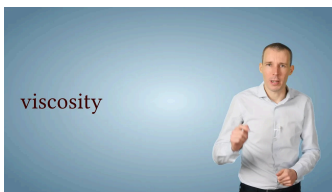
- depends on our own body temperature (hot water “feels” hotter when we are cold)
- depends on the heat capacity of the fluid (cold water “feels” colder than air at the same temperature)
- and depends on the amount of convection taking place (ambient air “feels” colder on a windy day).

Advice from an expert

Never make guesses about the dynamics of a flow using your own perception of temperature. The human body constantly rejects or absorbs heat, and makes for a very bad thermometer. Wind may just *feel* cold because the air movement increases heat transfer off your skin, and not because of changes in air pressure or in density. If needed, use a real thermometer!



1.6 Viscosity



Video: viscosity

by OC (STYL)
<https://youtu.be/aLorUltOvwg>

We have started in Section 1.2 by saying that a fluid can deform endlessly: it will never “snap” or break apart. However this deformation is not “for free”: it will require force and energy inputs which are not reversible (they are not reversed if the motion is reversed). Resistance to straining in a fluid is measured with a property named *viscosity*.^w

In informal terms, viscosity is the “stickiness” of fluids: for example, honey and sugar syrups are more viscous than water.

More formally, viscosity is quantified as follows. Imagine a small brick-shaped element of fluid, which is deformed (strained) horizontally, as shown in Figure 1.5. The continuous straining of the brick requires a force F per unit area A , called *shear stress* τ (see also section Section 1.8.2 further down). We expect that the shear increases with both the “stickiness” of the fluid —the viscosity— and with the speed Δv at which the brick is strained. Conversely, we expect that the shear will decrease when the element height Δy is increased.

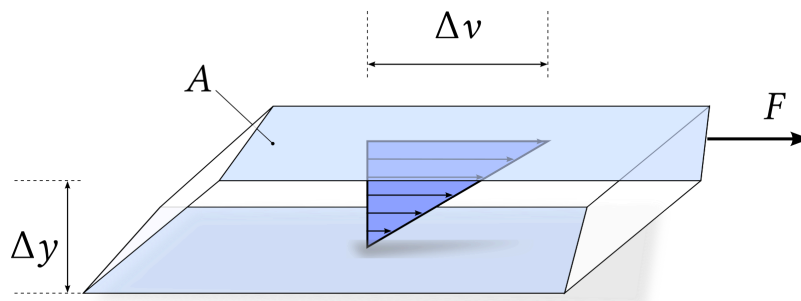


Figure 1.5: A brick-shaped element of fluid is strained, by applying a velocity difference Δv between the top and bottom surface. The horizontal force F required on the top edge, divided by the area A , is the shear stress τ (see also section Section 1.8.2 p. p. 8 further down). The higher the required shear stress for a given strain rate, the more viscous the fluid is.

Figure CC-0 Pm.schroeder/OC

We define the viscosity μ as the ratio between the required shear stress, $\tau = \frac{F}{A}$, and the rate at which the brick is strained, $\Delta \frac{v}{\Delta y}$:

$$\mu \equiv \frac{\tau}{\left(\frac{\Delta v}{\Delta y}\right)} \quad (1/3)$$

where μ is the viscosity (N s m^{-2} or Pa s);
 τ is the horizontal shear in Figure 1.5 (Pa);
 Δu is the velocity difference between top and bottom planes in Figure 1.5 (m s^{-1});
 and Δy is the height difference between top and bottom planes in Figure 1.5 (m).

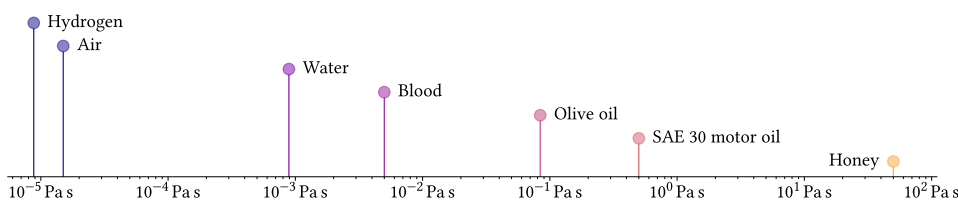


Figure 1.6: The value of viscosity for a few fluids. Pay attention to the logarithmic scale: honey on the right is about ten million times more viscous than hydrogen on the left.

Figure CC-BY-NC OC

The dimension of viscosity is in (force per area) divided by (velocity per distance), and so this turns out as Pascal-seconds in SI units. We will come back to shear and viscosity in chapter 3.

1.7 Mass flow and volume flow

Mass flow^w is noted $\dot{m}$ and represents the amount of mass flowing through a chosen surface per unit time. When the velocity across the surface is uniform, it can be quantified as:

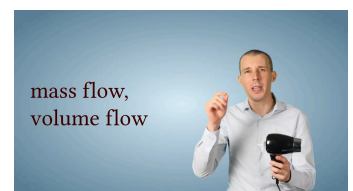
$$\dot{m} = \rho V_{\perp} A \quad (1/4)$$

where $\dot{m}$ is the mass flow (kg s^{-1});
 ρ is the fluid density (kg m^{-3});
 A is the area of the considered surface (m^2);
 and $V_{\perp}$ is the component of velocity perpendicular to the surface (m s^{-1}).

Instead of $V_{\perp} A$, we might pick $V A_{\perp}$: the velocity and the area of a surface *perpendicular* to that velocity:

$$\dot{m} = \rho V A_{\perp} \quad (1/5)$$

where V is the flow speed (m s^{-1});
 and $A_{\perp}$ is the area of a surface perpendicular to the flow velocity (m^2).



Video: mass flow, volume flow
 by OC (STYL)
https://youtu.be/SNCJ_LVNvK0

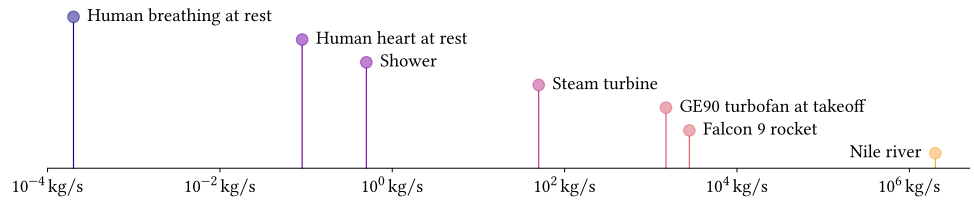


Figure 1.7: Approximate mass flows in various fluid flows.

Figure CC-BY-NC OC

Volume flow^w is noted $\mathcal{V}$ and represents the volume of the fluid flowing through a chosen surface per unit time. Much like mass flow, when the velocity is uniform, it is quantified as:

$$\mathcal{V} = V_{\perp} A = V A_{\perp} = \frac{\dot{m}}{\rho} \quad (1/6)$$

1.8 Fluids respond to things



Video: fluids respond to things
by OC (STYL)
https://youtu.be/ZQozc_E9WBI

Fluids are subjected to, and subject their surroundings and themselves to forces. Identifying and quantifying those forces allows us to determine how they will flow. Three types of forces are relevant in fluid dynamics: gravity, pressure and shear. Gravity (weight) is rather obvious, so let's take a look at the two others:

1.8.1 Pressure

We can approach the concept of **pressure^w** with the following conceptual experiment: if a flat solid metal plate is placed inside a moving fluid, then the pressure p will be the ratio of the perpendicular force $F_{\perp}$ to the surface area A :

$$p \equiv \frac{F_{\perp}}{A} \quad (1/7)$$

where p is the pressure (N m^{-2} or Pa , $1 \text{ Pa} \equiv 1 \text{ N m}^{-2}$);
 $F_{\perp}$ is the component of force perpendicular to the surface (N);
and A is the surface area (m^2).

The standard (SI) unit of pressure is the Pa ($1 \text{ Pa} \equiv 1 \text{ N m}^{-2}$), but practice it is often measured in bar ($1 \text{ bar} \equiv 1 \cdot 10^5 \text{ Pa}$) or . Ambient atmospheric pressure at normal altitude varies with the weather and is approximately 1 bar. There are other units too which we will see later, when we return to pressure in chapter 3.

1.8.2 Shear

In the same thought experiment as above, the **shear^w** noted τ (greek letter “tau”), expresses the efforts of a force *parallel* to a surface of interest:

$$\tau \equiv \frac{F_{\parallel}}{A} \quad (1/8)$$

where τ is the shear (N m^{-2} or Pa);
 $F_{\parallel}$ is the component of force parallel to the surface (N);
and A is the surface area (m^2).

Contrary to pressure, shear is not a scalar: it can (and often does) take different values in different directions: on the flat plate above, it would have two components and could be represented by a vector $\vec{\tau} = (\tau_x, \tau_y)$. We will explore more about shear in chapter 3.

1.9 Recognizing turbulence

Turbulence shows up everywhere in fluid mechanics, and so it is important you know how to recognize it, and that you understand what makes it “tick”. We’ll begin with a (dense) definition: *turbulence* is fluid motion in multiple scales which is chaotic and dissipative.

1.9.1 Chaos, not randomness

this content is well-covered in Leschziner [4]

Imagine pushing a bicycle and then leaving it to roll by itself. It will continue rolling and eventually fall to the side. Which side, left or right, depends on the initial conditions: even a minute modification to the start position is likely to influence the result. The fall is deterministic, and can be calculated very precisely, but with a very strong dependence on the initial conditions.

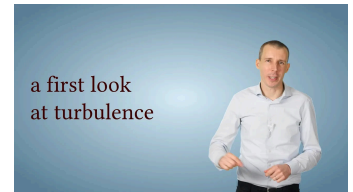
Turbulence is like that. Fluid motion follows laws which are fully deterministic (the Navier-Stokes equation, which we will explore in chapter 4), but in many situations, the exact patterns cannot be predicted because, just like the for the bicycle above, they depend very minutely on the initial configuration.



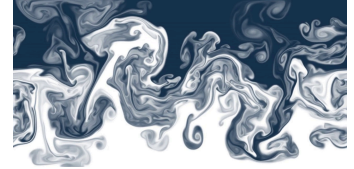
Figure 1.8: A column of hot air from burning incense rising through cold air is an unstable situation. After a certain length, the flow breaks down into chaotic patterns. The occurrence is predicted across all fluids, plume diameters, and velocities: every time, the Reynolds number, introduced below, is the determining parameter.

Photo CC-BY by Rafa Espada

Thus, in two identical turbulent flow experiments, the details of the flow will be different. In this sense, turbulent flow is *chaotic* (depending extremely sensitively on initial conditions) but not *random*: it remains predictable, governed by well-known deterministic laws in which chance does not play a role. The effective engineer or scientist will determine 1) what general characteristics of turbulence *do* remain identical in both flows, and 2) how they affect the main, global flow characteristics.



Video: a first look at turbulence
by OC (STYL)
<https://youtu.be/8WK2EbnQPY>



Video: simulation of two miscible fluids of different densities layered one on top of the other (color representing density). The “perfect” uniform initial situation is unstable and leads to chaotic (hard to predict) patterns whose details will depend strongly on minute changes in the initial conditions. (A 2D DNS simulation performed with MicroHH)
by Chiel van Heerwaarden (CC-BY)
<https://vimeo.com/84518319>



Video: turbulence and flow instability
by OC (STYL)
<https://youtu.be/uyosU1oYDr0>

1.9.2 Structures of all sizes

You will recognize turbulence in that it features flow structures of all sizes. We often call these structures *vortices* (plural of *vortex*) or sometimes *eddies*, but for now just think of blocks of fluids with a recognizable shape. A turbulent flow will always feature relatively large structures, which break down into ever smaller structures (we call this a *cascade* of vortices).

Not all complicated flows are turbulent. Well-known patterns such as a *von Kármán vortex street* (Figure 1.9) or a series of *wave clouds*, for example, are not turbulent. Those oscillations occur at a single recognizable frequency and scale, and once the phenomena has begun, we can predict their shape very well. Most surface waves on a body of water (e.g. waves in open sea, Figure 1.9) are not turbulence: although they may be partly chaotic, they dissipate very little energy (unless they crash on a shore) and propagate over very large distances.

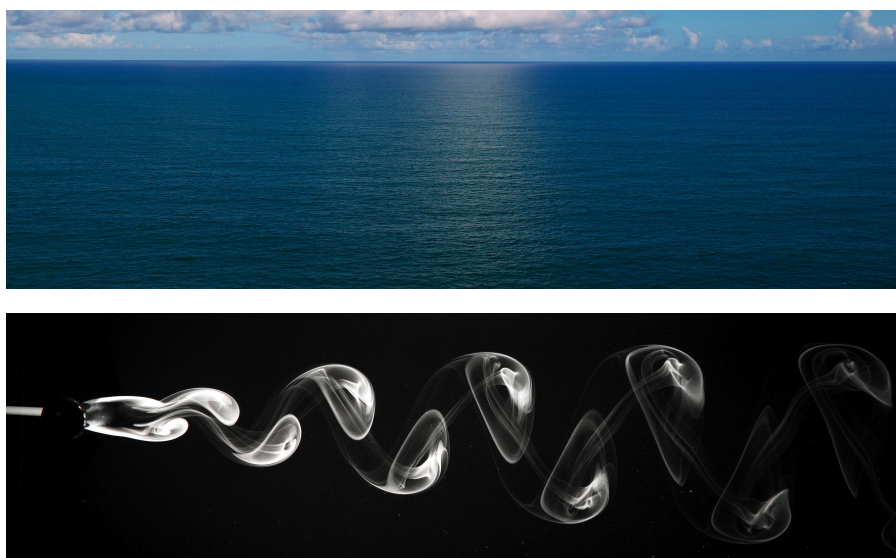


Figure 1.9: Not turbulence: surface waves on the ocean (top) are complex, but not fully-chaotic, and they feature very little dissipation. Well-known oscillatory patterns such as the *von Kármán vortex street* feature one dominant frequency and one dominant vortex size: they are not turbulent either.

Sea wave photo CC-BY-SA by Tiago Fioreze (cropped); Wake photo CC-BY-SA by Jürgen Wagner

Molecular motion is, at the macroscopic scale, completely random, and will appear in measurements as Gaussian white noise with no distinguishable range of frequencies, and no dissipation phenomenon: it is not turbulence, either.

1.9.3 Turbulence is dissipative

this content is well-covered in Davidson [3]

Turbulence occurs because shear, and sometimes also pressure, is applied non-uniformly in a fluid. Typically this will be behind an obstacle and along-side solid surfaces.

Turbulence is only sustained if shear is continually applied to the flow (for example by the walls of a long pipe). If no sustaining source is provided, turbulence decays and eventually dies. We say that turbulence is *dissipative*. Large-scale motion created by an initial obstacle continually breaks down into ever smaller-scale motion. Ultimately, the motion becomes so small and slow that the energy is dissipated by viscosity into molecular-scale motion.

In this sense, turbulence is not magic: it occurs in circumstances that are well understood and documented. Turbulence takes energy out of the main flow and ultimately transforms it into heat. It stops by all by itself.

1.10 Critical questions: time dependency

When a fluid dynamicist starts working on a fluid flow, one of the first questions they want to answer is: how long must they study the flow to make sure they describe it completely?

There is no way to calculate the answer: in practice we use observation and experience. The first step is to “erase” turbulence in our minds: visualize the flow in its averaged-out version without the chaotic contribution of turbulence. Then:

- If a single picture describes the flow: the flow is called *steady*. Steadiness is dependent on the chosen point of view: for example, the airflow around an aircraft in cruise flight is steady as seen from inside the airplane, but is highly unsteady from the point of view of the poor air particles directly in the path of the aircraft.
- If the “picture” changes with time, i.e. a video is needed to describe the flow, then it is called *unsteady*.

Advice from an expert

Don't confuse steadiness (the fields of properties remain the same as time passes) with *uniformity* (the properties are the same everywhere in space). Steadiness means the fluid dynamicist smiles because the problem does not change. Uniformity means the fluid dynamicist is bored because everything is moving exactly in the same direction.



Video: time dependency by OC (STYL)
<https://youtu.be/aLW9-W9-bUQ>

1.11 Critical questions: Turbulence

A question *every* fluid dynamicist has in mind right from the start is “is this flow going to be turbulent?”. There is a simple way to answer this: by calculating the *Reynolds number*. The only drawback is that this requires taking a leap of faith, because the *reasons* for why this works will only become clear later.

To determine whether the flow is turbulent, calculate the Reynolds number, which is noted [Re]:

$$[\text{Re}] \equiv \frac{\rho V L}{\mu} \quad (1/9)$$

where [Re] is the Reynolds number (no unit);
 ρ is the density of the fluid (kg m^{-3});
 V is one (averaged) velocity that describes the flow adequately (m s^{-1});
 L is the size of the largest flow structures you expect to observe in the flow (m);
 and μ is the viscosity of the fluid (Pa s).

We will see later (in chapter 5) that the inverse of the Reynolds number appears in front of the damping term in the fluid movement equations: the larger [Re], and the less damping there is, and the more likely the flow is to be turbulent.



Video: turbulence and how to calculate a Reynolds number, with an error inside (try to spot it!) by OC (STYL)
<https://youtu.be/pc5kO-ZZMHM>

In practice:

- if $[\text{Re}] < 100$ then you can expect the flow to be *laminar* (the opposite of turbulent);
- if $[\text{Re}] < 1000$ then you can expect the flow to be turbulent.

The Reynolds number does not quantify the *amount* of turbulence, but the *stability* of the situation: the *likeliness* that turbulence will occur. We are only interested in how many digits the Reynolds number has (nobody compares a Reynolds of 12000 with a Reynolds of 12004).

We will return to this concept, but in the meantime, adopt the reflex of always calculating $[\text{Re}]$ when getting started with a fluid mechanics problem. It does not cost much work, and it will build up your intuition about flows.

Advice from an expert

There definitely is something disquieting, even unse-
rious about how the Reynolds number is defined. “The
size in meters of the biggest flow structure I expect to
see”? Hello? But trust me, it will make sense once we
are deep in the math. Don’t wait until then to do what
every fluid dynamicist worth their salt does when first
approaching a problem. Calculate $[\text{Re}]$ every time to
build up your sense of what is big and small, and which
flows are going to be laminar or turbulent.

Also it will earn you street cred in the lab. When they want to compare
themselves to their peers (and maybe try to impress them), fluid dynami-
cists won’t want to know your flow speed or mass flow, but instead just
casually drop: “what’s your Reynolds number?”



1.12 Three big tools

Fluid mechanics is hard. Many science and engineering disciplines empower you to calculate things about the world, but a fluid mechanics course involves a lot of theory with little results because most fluid flows just are complicated. We can’t “calculate” swimming in a pool, blowing dust off a shelf, or stirring a cup of coffee with a pen and paper. They involve three-dimensional field of properties (pressure, temperature, etc.), one of which, velocity, is itself three-dimensional.

In practice fluid dynamicists have three big tools to investigate a fluid flow.

- **Experiment:** we go about measuring directly or indirectly the things we are after. This is expensive (in equipment and in time, because “fluid flow” is a lot of information). A good understanding of the physics is needed if we want to scale flows and extrapolate results.
- **Computer simulation:** we call this *Computational Fluid Dynamics*^w. Computers are good at manipulating large amounts of data in three dimensions. But fluid flow simulations are *incredibly slow* because the equations of fluid motion are solved only iteratively. This means that we spend a lot of time building all kinds of models and simplifications in simulations in order to get results in time. A good understanding of the physics is needed to make sure the simulations are meaningful.
- **Theory:** this is what this course is about. A good understanding of the physics and engineering of fluid flow will enable you to make smart decisions. What is going to happen in this flow? What kind of measurement



SMBC #2010-08-29: cooperation
between theoretical and
experimental scientists

by Zach Weinersmith
<https://smbc-comics.com/comic/2010-08-29>

would I need to check that? What is the influence of this parameter? If the computer calculated this flow, how would I check that the results make sense?

And so now with the fundamentals laid out, we can begin!

What ideas and equations do you think were the most important in this chapter?



Here is space reserved for a funny drawing about chapter 1

